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Persistent non-ergodic fluctuations in mesoscopic insulators: The NSS model in the unitary and symplectic ensembles

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Abstract. We give a detailed picture of the mesoscopic conductance fluctuations in the deep insulating regime (DIR) within the Nguyen, Spivak and Shklovskii model in the unitary and symplectic ensembles. Slutski's theorem is invoked to rigorously state the ergodic problem for conductance fluctuations in the DIR, in contrast with previous studies. A weakly decaying behavior of the log-conductance correlation function, even weaker when spin-orbit scatterers are included, is established on the relevant field scale of the model. Such a slow decay implies that the stochastic process, defined by the fluctuations of the log-conductance, is non-ergodic in the mean square sense in the ensembles with the reported symmetries. The results can be interpreted in terms of the effective number of samples within the available magnetic scale. Using the replica approach, we derive the strong localisation counterparts of the well known 'cooperon' and 'diffuson' which permit analyzing quantitatively the decaying behavior of the correlation function and reveal its symmetry related properties in agreement with the numerical results.

PACS. 71.23.An Theories and models; localized states – 71.70.Ej Spin-orbit coupling, Zeeman and Stark splitting, Jahn-Teller effect – 72.20.Ee Mobility edges; hopping transport – 74.40.+k Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.)

1 Introduction

The nature of fluctuations in both the metallic state [1,2], and in disordered insulators [3–5], have been a matter of interest for both theoretical and experimental studies. Whereas in the metallic regime the basic aspects of fluctuations have been elucidated, in the regime of hopping transport, the nature of fluctuations is still an open question. The deep insulating regime (DIR) where transport occurs *via* variable range hopping (VRH), is defined as the regime where the localization length is the smallest scale as compared to the elastic mean free path and hopping lengths, *i.e.* $\xi < \ell < t$ respectively [6]. Coherence effects are possible in this regime because phase breaking events occur at the hopping length [7], which is larger than ℓ. Important signatures of quantum interference in disordered insulators are the classic magneto-fingerprints, reproducible fluctuations in the conductance with magnetic field, and a low field positive magneto-conductance.

An important property of mesoscopic conductance fluctuations in the metallic phase is their ergodicity. At the mesoscopic level, the sample size is less than thermal diffusion length or the dephasing length, whichever is shorter, such that sample to sample fluctuations are visible and the system does not self-average. In the metallic regime, an ergodic criterion is verified whereby the magnetic field (or energy) induces conductance fluctuations equivalent to sample to sample fluctuations known as the Lee-Stone criterion [1]. In contrast, experimental results show that the DIR exhibits non-ergodicity in the log-conductance in the Lee and Stone sense [4,5,7,8], *i.e.*, the variance over samples is larger than the variance over field. The latter has been shown only in the absence of spin-orbit (SO) scattering since to our knowledge the ergodic criterion has not been tested in the presence of such impurities. Careful measurements of Fowler *et al.* [9], Ladieu *et al.* [4] and Orlov *et al.* [5] have shown that a) field fluctuations do not decorrelate disorder fluctuations, b) field fluctuations do not change the identity of the hop, c) the field average of the variance over the samples is larger than the sample average of the variance over the field and d) there exists a decorrelation field B_c determined by the decay of field correlation function, which defines an equivalent new sample.

In this work we first address the problem of fluctuations and the question of ergodicity in DIR within the Nguyen, Shklovskii and Spivak (NSS) model [7]. The question was first addressed numerically, within the NSS

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model, by its proponents who found violation of ergodicity in the Lee and Stone sense. We undertake the study of fluctuations from the point of view of the verification of Slutski's theorem concerning ergodicity. Such a theorem first entails specifying the stationarity of the analyzed fluctuations, a fact which has gone undiscussed in previous works, and can lead as we show here, to important hopping length dependent effects. The interest in studying ergodicity from this point of view is that it will clarify why the system is non-ergodic. We will show this can be understood in terms of the number of effective samples within the physical magnetic field range.

The replica-cummulant approach developed in reference [10] is the only current analytical method that correctly assesses correlations of the NSS model. Scaling of fluctuations have been computed from such a scheme [10] but persistent (field strength independent) fluctuations where only recently derived by Medina *et al.* [11]. Here we extend the method to compute conductance correlation functions within both the unitary and symplectic ensembles. This approach yields new predictions to be tested experimentally such a symmetry dependent decay length in the magnetic field. Especially intriguing is the nature of fluctuations in the symplectic ensemble, where no cooperon correction exists, while the diffuson component is non-zero.

Summarizing our paper, we first present Slutski's theorem with some detail in Section 2. In Section 3, we describe the NSS model and define the random processes we analyze. In Section 4, we analyze the fluctuations predicted by the NSS model, with and without SO scattering, using the criteria described in Section 2 and rigorously verify, numerically, the non-ergodic behavior of fluctuations. In Section 5, within the replica-cummulant approach, we derive the cooperon and diffuson analogs in strong localization, which we use to quantitatively explain the decaying behavior of the numerical conductance correlation function. Finally we conclude by discussing our results and pointing out some experimental implications.

2 Ergodicity of transport fluctuations

Given a physical quantity $F(\mathcal{H},B)$ depending on the disordered Hamiltonian $\mathcal{H}$ and magnetic field B, we denote by $\overline{F(\mathcal{H},B)}$ the sample to sample average, or disorder average, and by $\langle F(\mathcal{H},B)\rangle = \Delta B^{-1}\int_{B_i}^{B_f} \mathrm{d}B\ F(\mathcal{H},B)$, the field average for a given sample or disorder realization. In order to be able to estimate the sample average from the field average of a given sample the following conditions must be satisfied: a) $\lim_{B\to\infty}\sigma_{mss}(B) = \overline{[F(\mathcal{H},B)-\langle F(\mathcal{H},B)\rangle]^2} \to 0$ and b) $\overline{F(\mathcal{H},B)} = \overline{\langle F(\mathcal{H},B)\rangle}$. The verification of both conditions is known as *ergodicity in the mean square sense* (m.s.s.), or the random function $F(\mathcal{H},B)$ is said to be ergodic in the mean-square limit [12]. The condition $\overline{F(\mathcal{H},B)} = \overline{\langle F(\mathcal{H},B)\rangle}$ expresses global stationarity, which means that these averages are independent of B. One can cast conditions a) and b) into a single statement on the disorder fluctuations of the field average, *i.e.* $\lim_{B\to\infty}\sigma_{mss}(B) = \lim_{B\to\infty}\mathrm{Var}_d(\langle F(\mathcal{H},B)\rangle) = \overline{[\overline{\langle F(\mathcal{H},B)\rangle} - \langle F(\mathcal{H},B)\rangle]^2} \to 0$, where Var_d denotes the variance over disorder [13]. This property implies that for one realization of disorder there are enough 'equivalent samples' within the magnetic scale, such that the average in the field, does not depend, statistically, on the particular realization.

One could also ask to make estimates of $\mathrm{Var}_d(F(\mathcal{H},B))$ from $\mathrm{Var}_B(F(\mathcal{H},B))$, (here $\mathrm{Var}_B(F(\mathcal{H},B))$ means the variance over the field B of $F(\mathcal{H},B)$), or more generally, to make an estimate of a function of the basic process $F(\mathcal{H},B)$, $g(F(\mathcal{H},B))$. The necessary and sufficient conditions such that one can estimate $\overline{g(F(\mathcal{H},B))}$ from one realization of disorder with the field average $\langle g(F(\mathcal{H},B))\rangle$, is dictated by slutsky's theorem. One can write $\sigma_{mss}(B_f) = \lim_{B_f\to\infty}\frac{2}{\Delta B^2}\int_{B_i}^{B_f} dB(B_f-B)C(\ g(F(\mathcal{H},B)))$, with

$$C(g(F(\mathcal{H},B,\Delta B))) = \overline{\Delta g(F(\mathcal{H},B+\Delta B))\Delta g(F(\mathcal{H},B))}, \tag{1}$$

where $C(g(F(\mathcal{H},B,\Delta B)))$ is the correlation function for $g(F(\mathcal{H},B,\Delta B))$ and $\Delta g(F(\mathcal{H},B+\Delta B)) = g(F(\mathcal{H},B+\Delta B)) - \overline{g(F(\mathcal{H},B+\Delta B))}$. One can easily realize that a strong decaying behavior of the correlation function with correlation field $B_c \ll B_f$ will be a sufficient condition for ergodicity in the m.s.s., *i.e.* $\sigma_{mss}(B_f)\to 0$. In fact this is also a necessary condition.

For the above theorem to hold, stationarity must be assumed. Stationarity implies that $\overline{F(\mathcal{H},B)}$ does not depend on B and $C[g(F(\mathcal{H},B,\Delta B))]$ depends only on ΔB [14]. In this work we are interested in testing the ergodicity in two cases: $g(X)=X$ for the average and $g(X)=\overline{X^2}-\overline{X}^2$ for the variance. The first case corresponds to the usual meaning of ergodicity in statistical mechanics. The second case, usually named the Lee-Stone criterion, refers to the equivalence of sample to sample fluctuations and magnetic field fluctuations [1,12,15].

3 The NSS model

We now examine the fluctuations in the DIR within the Nguyen *et al.* [6,7] model (NSS). The NSS model's crucial insight is that coherence is maintained within a Mott hopping length, where the conductance is a sum of coherent *forward directed Feynman paths* which interfere which each other. The NSS model describes the quantum behavior of the critical (bottleneck) hop in the Miller-Abrahams network [16]. The existence of many randomly oriented critical hops tend to average the macroscopic conductance, eliminating fluctuations [9]. Here, we focus on the low temperature regime where critical hops do not trivially self average [8,17], *i.e.*, the percolation correlation length ξ_p is such that $\xi_p = \xi(T_0/T)^{(\nu+1)/(D+1)} \sim L$, where ν is the percolation correlation length exponent, D is the spatial dimension and T_0 a disorder parameter. This is the mesoscopic regime [8].

In the two dimensional NSS model, impurities are placed on the sites of a lattice of main diagonal length t (the hopping length, $t = \xi(T_0/T)^{1/(D+1)}$, Mott's law) and lattice spacing ℓ. We apply a magnetic field B, perpendicular to the plane, changing only the phases of the electron paths. The overall tunneling amplitude is computed by summing all forward directed paths between two diagonally opposed points, each contributing an appropriate quantum mechanical complex 2×2 matrix weight given by the Hamiltonian

$$\mathcal{H} = \sum_i \epsilon_i a^\dagger_{i,\sigma} a_{i,\sigma} + \sum_{\langle ij \rangle_{\sigma,\sigma'}} V_{ij,\sigma\sigma'} a^\dagger_{i,\sigma} a_{j,\sigma'}, \qquad (2)$$

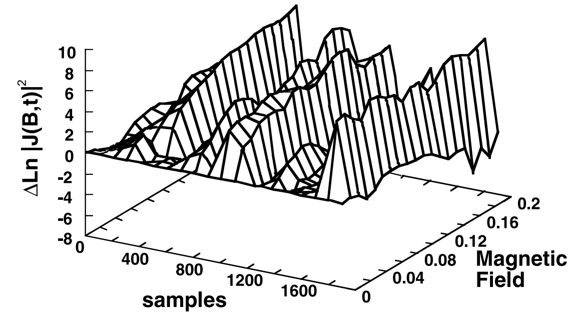
where ϵ_i is the site energy, and $V_{ij,\sigma,\sigma'}$ represents the nearest neighbor couplings or transfer terms which includes a randomly chosen SU(2) matrix describing a spin rotation due to SO scattering. Within the NSS model, we choose site energies to be $\epsilon_i = \pm W$ with equal probability [7,10]. Without SO, or *unitary* ensemble, the coupling terms are diagonal in spin space $V_{ij,\sigma\sigma'} = V_{ij}$, and the Green's function between the initial and final site is given by

$$\langle i|G(E)|f\rangle = \left(\frac{V}{W}\right)^t J(B,t);$$
$$J(B,t) = \sum_{\Gamma'}^{directed} \prod_{i_{\Gamma'}} \eta_{i_{\Gamma'}} \mathrm{e}^{\mathrm{i}\phi_{i_{\Gamma'}}}, \qquad (3)$$

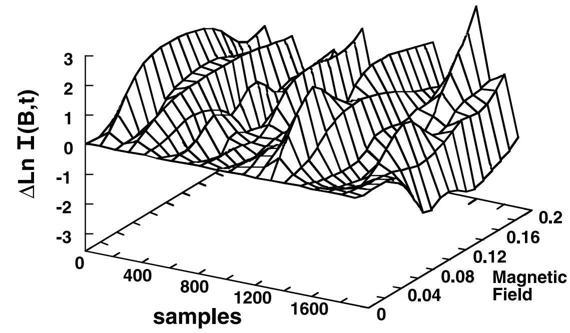
where $\phi_{i_{\Gamma'}}$ is the phase gained through path $i_{\Gamma'}$ due to the magnetic vector potential, and Γ' labels all directed paths that go from i to f through the lattice and $\eta_i = \mathrm{sign}\,(\epsilon_i) = \pm 1$ [18]. In the presence of SO scattering, or *symplectic ensemble*, the Green's function is the 2×2 matrix

$$J(B,t) = \sum_{\Gamma'}^{directed} \prod_{i_{\Gamma'}} \eta_{i_{\Gamma'}} U_{i_{\Gamma'}} \mathrm{e}^{\mathrm{i}\phi_{i_{\Gamma'}}}. \qquad (4)$$

The Green's function consist of a sum of terms, one per path, each being a product of random numbers, random SU(2) matrices U, and a deterministic disorder independent phase factor from the magnetic vector potential [19]. The complex function $J(B,t)$ (a complex matrix function in the presence of SO) contains the interference information including correlations due to crossing of paths, and the factor $(V/W)^t$ is the leading contribution to the exponential decay of the localized wavefunction. We use the transfer matrix approach in order to compute $J(B,t)$, exactly, for each realization of disorder [10]. The random processes in question represent the log-conductance; they are $F(\mathcal{H}, B) = \ln(J^\dagger(B,t)J(B,t))$ in the unitary case and $F(\mathcal{H}, B) = \ln(I(B,t))$, in the symplectic case, where $I(B,t) = (1/2)\mathrm{Tr}(J^\dagger(B,t)J(B,t))$ [10]. In this work we compute the magnetic field B or changes in magnetic field ΔB in flux units ϕ_0/ℓ^2, where ϕ_0 is the flux quantum.



(a)



(b)

Fig. 1. Conductance fluctuations of a) $\ln|J(B,t)|^2$ (unitary) and b) $\ln|I(B,t)|$ (symplectic) as a function of sample number and magnetic field, for two dimensional systems of hopping length $t = 15$. Field fluctuations do not decorrelate sample fluctuations and are evidently smaller. Fluctuations for the symplectic ensemble are notably slower than in the unitary ensemble as discussed in the text.

4 Fluctuations and ergodicity

4.1 Unitary ensemble

Figure 1a shows typical fluctuations of the log-conductance as a function of the sample number and the magnetic field in two dimensions. The figure clearly shows a mean increase of $\ln|J(B,t)|^2$ that dominates the fluctuations, *i.e.*, the mean behavior is visible for a single sample. It has been shown that $\overline{\ln|J(B,t)|^2}$ first increases proportional to B^2, crossing over to a slower growth as $B^{1/2}$ dictated by the magnetic length $\Delta B < B_c = \pi c\hbar/(\xi^{1/2} e\ t^{3/2})$ [10,20,21], therefore the process is not stationary in the field. However, in the higher field regime of slow growth, one finds a field above which the process can be considered as quasi-stationary, for all practical purposes, in the same fashion as has been considered in the metallic regime. In this range, while the log-conductance tends to saturate, the fluctuations persist as in mesoscopic fluctuation theory of metals [1]. Furthermore, we note that the average behavior is periodic in half the flux quantum ϕ_o per ℓ^2. This periodicity reveals an average field coupling to $2B$ [7] which has been demonstrated theoretically [11]. In three dimensions, the fluctuations are appreciably larger than the average behavior, and persistent fluctuations beyond the average conductance saturation field are also observed. The existence of such persistent fluctuations were first surmised by Sivan *et al.* [9,22] and Zhao *et al.* [20]. One very visible feature of Figure 1a, is the fact that field fluctuations do not decorrelate the disorder fluctuations, in agreement with the experimental findings of reference [5]. This suggests, non-ergodic behavior.

Following the concepts of section two, we analyze the ergodicity of the log-conductance fluctuations more carefully. To achieve this we have to check two issues: first, whether stationarity is reasonably fulfilled, and second to verify the ergodicity condition and the decaying behavior of the correlation function. Figure 2a shows the quantities $\mathrm{Var}_d(\overline{\langle F(\mathcal{H},B)\rangle})$ (circles) and $\overline{[\overline{F(\mathcal{H},B)} - \langle F(\mathcal{H},B)\rangle]^2}$ (diamonds) where $F(\mathcal{H},B) = \ln|J(B,t)|^2$. The field averaging interval is $[B_i, B_f]$. In this figure, as the field B_i is increased from zero to 0.1 in $[\phi_0/\ell^2]$ units, the computed variance is reduced (diamonds) until a saturation initial field is achieved $B_i = B_s$. $\mathrm{Var}_d(\langle F(\mathcal{H},B)\rangle)$ (circles) is computed at B_s. The overlap between both averages occurs sooner for smaller hopping length t. Figure 2a contains information that is twofold: First we confirm effective stationarity above field $B_s \sim 0.1\phi_0/\ell^2$ *i.e.* $\overline{\langle F(\mathcal{H},B)\rangle)} = \overline{F(\mathcal{H},B)}$, and second, that evidently $\mathrm{Var}_d(\langle F(\mathcal{H},B)\rangle)$ does not converge to zero contrary to the ergodicity criterion in the *m.s.s.* The field above which the process can be considered quasi-stationary, B_s, increases with the hopping length. This feature is of importance for calculating other relevant quantities, as any question on ergodicity presumes at least quasi-stationarity.

To further substantiate this result, we compute the correlation function. Figure 3 shows the normalized correlation function for both unitary and symplectic cases, $t = 30$ and $B_i = B_s$. One sees a weakly decaying behavior on the physical field scale $[1\ \phi_0/\ell^2]$, with a characteristic decay field B_c defined as the field at which the correlation decreases to half its initial value. Note the initial downward curvature of the correlation, which then turns to an upward curvature whose origin will become clear within replica theory.

The basic finding is then that, the slow decaying behavior of the correlation function precludes the construction of enough 'effective samples' (number of $\Delta B = B_c$ ranges within the physical field scale) from the field fluctuations and therefore non-ergodic behavior is established. To illustrate this point, we depart from $\sigma_{mss}(B_f) = \frac{2}{(B_f-B_i)^2}\int_{B_i}^{B_f} \mathrm{d}B[(B_f - B)C(F(\mathcal{H},B))]$; in order to have enough effective samples in the field scale within the validity of the model ($B_f \leq 1\phi_0/\ell^2$), there should be a decorrelation field $B_c \ll 1$, such that a large number of samples can be defined. This would imply $\sigma_{mss}(B_f) \approx 0$, clearly not the case from our results. On the other hand, it follows that the condition for ergodicity of the variance, which in general requires a stronger decaying behavior of the correlation function for process $F(\mathcal{H},B)$, is not fulfilled [12]. Therefore, one should expect that the Lee and Stone crite-

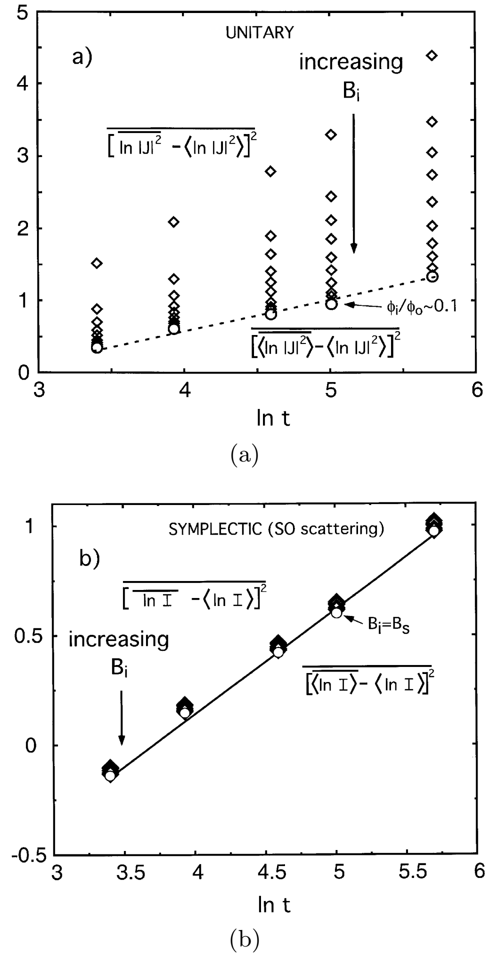


Fig. 2. Quasi stationary behavior in the unitary and symplectic ensembles. a) As the field B_i is increased, the average indicated (top, diamonds) converge to the dotted curve, implying stationary behavior and an increasing value of the average with the hopping length (non-ergodicity). The limiting B_s is $0.1\phi_0/\ell^2$. b) The symplectic case where convergence as a function of B_i is almost immediate, indicating stationarity.

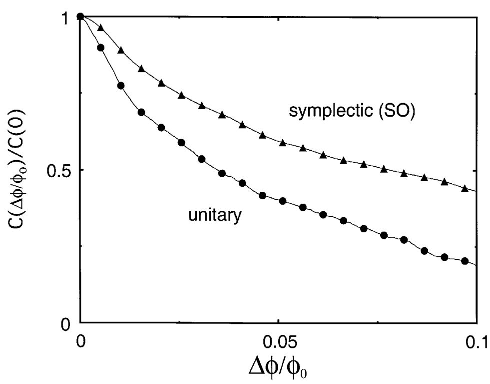


Fig. 3. Normalized correlations functions $C(B, \Delta B, t)$ for the unitary and symplectic ensembles as a function of ΔB for a given value of $B_i = 0.1$ (no further changes for higher B_i) and $t = 30$. The correlation functions initially decay with a downward curvature described by replica theory. Note that the decay for the unitary case is almost exactly twice as fast as in the symplectic case.

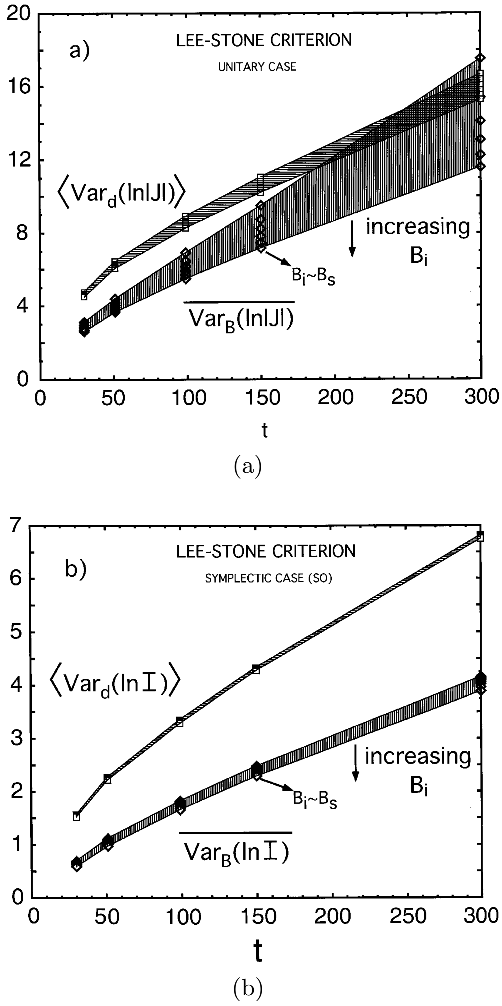


Fig. 4. The figure depicts the variance as defined by equations 5 and 6 as a function of t for a) the unitary ensemble and b) the symplectic ensemble. The effect of changing the values of B_i from 0 to 0.1 (indicated by the hatched regions) is clearly seen until saturation is attained at $B_i = B_s$. For the unitary case, B_i below the saturation field can suggest apparently ergodic ranges for large enough hopping lengths. Such fictitious crossing does not arise in the symplectic ensemble where non-ergodicity in the Lee and Stone sense is more marked.

rion of ergodicity on the relative magnitude of the field and sample fluctuations is not realized. We have compared the magnitude of the variance in field and sample to sample fluctuations. The idea of further averaging over disorder and over the field, is the same as in statistical mechanics using different initial conditions to improve the statistics. Figure 4a shows the averages

$$\langle \mathrm{Var}_d(\ln|J(B,t|)\rangle = \left\langle \overline{(\ln|J(B,t| - \overline{\ln|J(B,t|})^2} \right\rangle, \quad (5)$$

$$\overline{\mathrm{Var}_B(\ln|J(B,t|)} = \overline{\langle(\ln|J(B,t)| - \langle\ln|J(B,t)|\rangle)^2\rangle}. \quad (6)$$

The figure depicts the dependencies of expressions in equation (5) (squares and horizontal hatch) and 6 (diamonds and vertical hatch) on the hopping length and increasing B_i. Note that for $B_i \leq B_s$ ergodicity is apparently achieved (in the Lee and Stone sense) as the size increases. Nevertheless, if the variances are computed correctly ($B_i \geq B_s$) the absence of ergodicity is established for all hopping lengths studied. We find the characteristic B_i dependence shown in Figure 2 shows up as a crossing of both types of averages. There is again a clear tendency to saturate as B_i increases whereas the difference of the saturated averages widens with increasing t.

4.2 Symplectic ensemble: SO scattering

When SO scattering is present, the Hamiltonian exhibits symplectic symmetry. In two and three dimensions the fluctuations are markedly stronger than the average behavior. This fact is due to the absence of exponential corrections to the magnetoconductance in the symplectic ensemble for the NSS model [10]. The fluctuations are also less sensitive to field changes in stark contrast with the sharp changes in the absence of SO. This peculiarity will be understood in the next section. As in the unitary case, Figure 1b shows that field fluctuations do not decorrelate the disorder fluctuations. This finding has yet to be tested experimentally.

In the symplectic ensemble we are interested in $F(\mathcal{H}, B) = \ln|I(B,t)|^2$. Figure 2b shows immediate effective stationarity of the process $\ln(I(B,t))$ essentially independent of B_i and t. $\mathrm{Var}_d(\langle F(\mathcal{H}, B)\rangle)$ does not tend to zero with increasing B_f; on the contrary, it increases with a power of t [11], an indicator of non-ergodic behavior in the *m.s.s.* and there is no self-averaging as one increases the hopping length t (see Kramer and Mackinnon [2]). Figure 3 shows the correlation function for $B_i \geq B_s$ whose decay is weaker than in the unitary case, with a more rapid decay as t increases. Note that the correlation function for the symplectic case decays to half its height in twice the range as compared to the unitary case. This is a symmetry related effect borne out within the replica approach in the next section.

In view of the relative independence of Var_d on B_i in the symplectic case, non-ergodic behavior is pronounced no matter the range of fields considered. Thus, from the Lee and Stone criterion, we also find an independence of B_i (see Fig. 4b), with an increasing non-ergodicity with the hopping length. Such clearcut results should be observed in experimental samples.

5 Correlation function: the cooperon and diffuson

In this section we derive the cooperon and diffuson in the context of strong localization. These objects are used to explain, semi-quantitatively, the non-ergodic behavior in the mean square sense and the relative magnitude of the field and sample fluctuations found in experiments. For this purpose, the following relation is straightforwardly derived

$$\mathrm{Var}_d[F(B+\varDelta B,t)+F(B,t)] = \mathrm{Var}_d[F(B+\varDelta B,t)] + \mathrm{Var}_d[F(B,t)] + 2C\,[F(B,\varDelta B,t)]\,. \quad (7)$$

This relation is valid for both processes $\ln|J(B,t)|^2$ and $\ln I(B)$. For convenience, the composite process inside the brackets $(F(B,t))$ of equation (7) is denoted by $P(B,\varDelta B,t) = \ln|J(B+\varDelta B)|^2 + \ln|J(B)|^2$, and with SO by $P_{spinor}(B,\varDelta B,t) = \ln I(B+\varDelta B) + \ln I(B)$, such that the left hand side of equation (7) is in each case $\mathrm{Var}_d[P(B,\varDelta B,t)]$ and $\mathrm{Var}_d[P_{spinor}(B,\varDelta B,t)]$, respectively.

In previous work it has been shown that in the NSS model $\mathrm{Var}_d(\ln|J(B,t)|^2)$ is reduced with the field concomitantly with an increase in the conductance [10]. On the metallic side, an analogous behavior has been described which identifies two fundamental contributions to the field effect: the cooperon and the diffuson [1,2]. These contributions can be distinguished by the way they enclose the magnetic flux; while the cooperon is sensitive to $(2B+\varDelta B)$, the diffuson only responds to field changes $\varDelta B$. In the insulating regime, a mechanism similar to the cooperon, which saturates, is here associated with a positive magneto-conductance (MC). The latter, has been observed as a general effect [2,5,8,23,24]. A semi quantitative explanation for the behavior of the functions $\mathrm{Var}_d[P(B,\varDelta B,t)]$ and $\mathrm{Var}_d[P_{spinor}(B,\varDelta B,t)]$ can be found with the help of the cooperon and diffuson analogs. To achieve this goal, we consider the moments of process of $P(B,\varDelta B,t)$ and $P_{spinor}(B,\varDelta B,t)$. For the unitary case they can be generated by $\overline{[J^*(B+\varDelta B)J(B+\varDelta B)J^*(B)J(B)]^n}$. Recall, from equations (3) and (4), that this product can be visualized as a set of $4n$ paths [10], each one defined by the respective term in the product.

In order to have nonzero contributions after disorder average, the paths must pair up, as a consequence of the chosen distribution of the energies [25]. Neutral paths (field independent) are formed by pairing J^* and J at the same field (phase cancels). On the other hand, charged paths (field sensible) are formed by pairing either $J^*(B+\varDelta B)$ and $J(B)$ or $J^*(B+\varDelta B)$ and $J^*(B)$. In the absence of paired path intersections, self interference kills charged paths (their contribution decays exponentially fast). Nevertheless, if intersections are considered, one can have path exchanges for short distances, yielding a magnetic field coupling which is the source of the initial decay of the correlation function. There are three possible diagrams at a paired path crossing, the two that are field coupling, are depicted in Figure 5. Without SO, the spin indexes can be ignored, and one obtains [11]: a) one partner from $J^*(B+\varDelta B)$ pairs with one from $J^*(B)$, while one from $J(B+\varDelta B)$ and one from $J(B)$ follow a different path. Such a combination encloses $(2B+\varDelta B)$ and is therefore called *cooperon*-like. b) One partner is taken from $J^*(B+\varDelta B)$ and the other from $J(B)$ on the same path, while one $J(B+\varDelta B)$ and $J^*(B)$ follow another. Such a combination encloses only $\varDelta B$ and is called *diffuson*-like. Finally, one can have combination c) where one partner comes from $J^*(B+\varDelta B)$ and the other from $J(B+\varDelta B)$, leaving $J^*(B)$ and $J(B)$ to pair up. The latter combination is called uncharged and is immune to the field. Note that all previous cases satisfy overall neutrality so that the contributions are real as expected. The contribution of the replica cooperon and diffuson are the same at zero field and there is an additional contribution from the uncharged diagram. Further progress is achieved using the replica cummulant method.

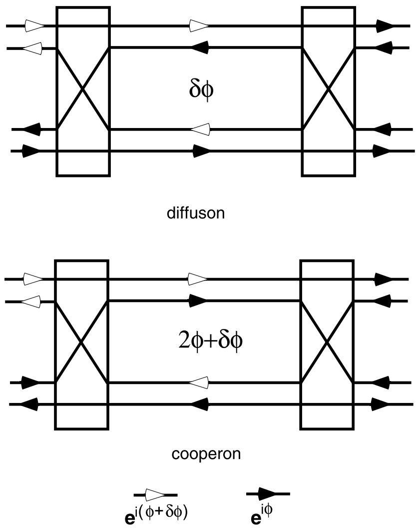


Fig. 5. The cooperon and diffuson diagrams in the unitary case discussed in the text. The cooperon is responsible for exponential corrections to the MC but saturates rapidly in the field. The diffuson is responsible for persistent field fluctuations independent of the magnitude of the field.

The replica-cummulant argument [25], maps the $4n$th moment problem onto the problem of $2n$ bosons with contact interactions. These interactions renormalise due to the diagrams above, making path intersections field dependent [10]. The $2n$ boson system with contact interactions according to the Hamiltonian $\mathcal{H}_{2n}$ [25] can be solved using the Bethe ansatz and has ground state energy $\epsilon^0_{2n} = \ln 4^n + \rho(B,\varDelta B)2n(4n^2-1)$, where $\rho(B,\varDelta B)$ is defined by

$$\overline{[J^*(B+\varDelta B)J(B+\varDelta B)J^*(B)J(B)]^n} = \mathrm{Tr}\,[\exp(-\mathcal{H}_{2n}t] \sim \exp(-\epsilon^0_{2n}t) = A(B,\varDelta B)\exp(\ln 4^n + \rho(B,\varDelta B)2n(4n^2-1)t), \quad (8)$$

valid at fixed n asymptotically for $t \to \infty$. On the other hand, the $4n$th moment can be expressed as a cummulant expansion valid at fixed t asymptotically for $n \to 0$, $\overline{[J^*(B+\varDelta B)J(B+\varDelta B)J^*(B)J(B)]^n} = \exp\sum_i \frac{(2n)^i}{i!}C_i[P(B,\varDelta B,t)]$, where $C_i[P(B,\varDelta B,t)]$ are the cummulants of process P. The subtleties concerning both limits have been discussed by Kardar [25]. Equating

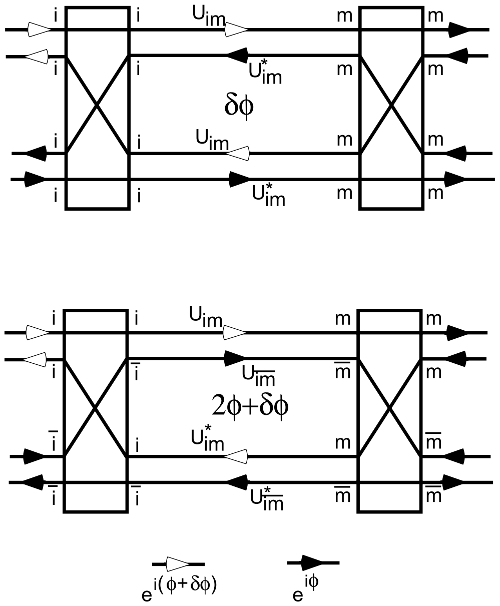


Fig. 6. The diffuson and the cooperon in the symplectic case. Here spin indexes are involved with the rules discussed in the text. The cooperon diagram vanishes indicating no exponential corrections to the MC. Diffuson contributions survive giving persistent conductance fluctuations in the field.

powers of n in the expressions above we obtain

$$\mathrm{Var}_d[P(B,\Delta B,t)] = [\rho_{coop}(2B+\Delta B) + \rho_{diff}(\Delta B)]\, t^{2/3} + \ln A(B,\Delta B). \quad (9)$$

Here $\rho(B,\Delta B) = \rho_{coop}(2B+\Delta B) + \rho_{diff}(\Delta B)$, were we have separated the path interaction in terms of the cooperon and diffuson contributions. The $t^{2/3}$ dependence has been previously checked numerically as the asymptotic behavior [10]. Beyond the saturation field of the average log-conductance, the cooperon term on the right hand side is a constant (the same for the variance on the right of equation (7) which depends on B) and the correlation function only depends on ΔB.

In the case of SO, it can be shown[10], that the only non-zero paired averages are $\overline{U_{\alpha\beta}U^*_{\alpha\beta}} = \frac{1}{2}$, $\overline{U_{\uparrow\uparrow}U^*_{\downarrow\downarrow}} = \frac{1}{2}$, $\overline{U_{\uparrow\downarrow}U^*_{\downarrow\uparrow}} = -\frac{1}{2}$, thus SO averaging brings a factor of $(\frac{1}{2})^2$ and forces the neutral paths to have parallel spins, while the spin of the two partners of charged paths must be antiparallel (see Fig. 6). As a consequence, one finds the cooperon diagrams cancel in pairs as concluded for the case of the magneto-conductance [10], and no exponential corrections to the conductance occur due to the cooperon. On the other hand, the diffuson is non vanishing and there are two possible combinations for incoming spin indices (all up and all down), such that in this case $\mathrm{Var}_d[P_{spinor}(B,\Delta B,t)] = \rho^{spinor}_{diff}(\Delta B)t^{2/3} + \ln A(B,\Delta B)$. This nice result explains why there are large fluctuations in the presence of SO, even if there are no exponential corrections to the magneto-conductance. For large ΔB and t, one finds the ratio of the variances approaches $\rho_{diff}(\Delta B))/\rho^{spinor}_{diff}(\Delta B) = 2$ $(= 1/(2\times(1/2)^2))$, in agreement with the numerical results.

From equation (7) one obtains

$$C(B,\Delta B,t) = \frac{1}{2}[(\rho_{coop}(2B+\Delta B) + \rho_{diff}(\Delta B)) - (\rho_{mc}(B+\Delta B) + \rho_{mc}(B))]t^{2/3} + S(B,\Delta B), \quad (10)$$

where $S(B,\Delta B) = (1/2)\ln A(B,\Delta B)/(\ln A'(B+\Delta B)\ln A'(\Delta B))$, are logarithmic corrections from the prefactors. $\rho_{mc}(B)$ defines the magneto-conductance from the analogous expressions, $\overline{[J^*(B)J(B)]^n} = \exp\{\sum \frac{n^i}{i!}C_i[lnJ^*(B)J(B)]\} = A(n,B,\Delta B)\exp(n\ln 2 + \rho_{mc}(B)n(n^2-1)t)$ [10].

From equation (10), one can gain qualitative and quantitative understanding of the decaying behavior of the correlation function for small ΔB. With the explicit field dependence of the cooperon and diffuson given by

$$\rho_{fluctuations} = \rho_{coop} + \rho_{diff} = (2\cos((2B+\Delta B)/B_c) + 2\cos(\Delta B/B_c) + 1)\rho(B=0), \quad (11)$$

and

$$\rho_{mc}(B) = [(2\cos(B/B_c)) + 1]\rho(B=0), \quad (12)$$

one obtains after the cooperon contribution has died out ($B > B_s$)

$$C(B,\Delta B,t) = [2\cos(\Delta B/B_c) - 1]\mathrm{Var}_d[F(0,t)]t^{2/3}. \quad (13)$$

Here, we have ignored the logarithmic corrections from equation (10). For $B=0$ and $\Delta B = 0$ one has $C(B=0,\Delta B=0,t) = \mathrm{Var}_d[F(0,t)] = \mathrm{Var}_d[F(0,t)] = \rho(B=0,\Delta B=0)t^{2/3}$, in accordance with equation (7). For $\Delta B < B_c$ one finds

$$C(\Delta B,t) = \left[1 - 2(\Delta B/B_c)^2\right]\mathrm{Var}_d[F(0,t)]t^{2/3}. \quad (14)$$

This behavior is qualitatively borne out in Figure 3, where one observes first a downward curvature for small ΔB. For a given ΔB one observes a faster decay with increasing t in accordance with $B_c = \pi c\hbar/(\xi^{3/2}et^{3/2})$[6] (not shown in the figure). This is also evident in the symplectic case where one has

$$C_{spinor}(B,\Delta B,t) = \frac{1}{2}\left[(\rho^{spinor}_{diff}(\Delta B)\right]t^{2/3}, \quad (15)$$

since ρ^{spinor}_{mc} vanishes [10] for the same reason that there is no cooperon contribution, so

$$C_{spinor}(B,\Delta B,t) = \frac{1}{2}(\cos(\Delta B/B_c))\rho(B=0)t^{2/3}. \quad (16)$$

In the $\Delta B < B_c$ limit we obtain

$$C_{spinor}(B,\Delta B,t) = (1 - (\Delta B/B_c)^2)\rho(B=0)t^{2/3}. \quad (17)$$

As found in Figure 3, the correlation function for the symplectic case shows a slower decrease, approximately by a factor of two, of its correlation function as predicted by the expressions above. Interestingly this a reflection of the symmetry. This fact also makes for an enhanced non-ergodicity for the symplectic case, because there are even less effective samples within the available field range. Finally, the full form of the correlation function will necesarilly need to account for a distribution of intersection diagram areas. Therefore, intersections with smaller and less probable areas determine the long ΔB behavior of the correlation function. Such a complete description has yet to be formulated.

6 Discussion and conclusions

Summarizing the results of this work, we have found that the log-conductance in the DIR of mesoscopic samples is non-ergodic in the mean square sense, and the fluctuations of the log-conductance are non-ergodic in the sense that sample to sample fluctuations are larger than magnetic field fluctuations. We have carefully assessed the importance of stationarity of fluctuations in determining ergodicity. In the absence of SO scattering, due to the exponential corrections to the magnetoconductance [10,11], only quasi stationarity can be achieved for fields larger than $B_s(t)$, whereas with SO, the small magneto-conductance and its rapid saturation [10,21], render $B_s(t)$ small, so that the process in the symplectic case can be regarded as effectively stationary independent of t. The field $B_s(t)$ is obtained numerically by varying B_i until one observes that $\sigma_{mss}(B_f) \approx \mathrm{Var}_d(\langle F(\mathcal{H}, B)\rangle)$. It is only when the averaging interval is taken as $[B_s(t), B_f]$, *i.e.*, when quasi-stationarity is achieved, that the proper comparison of the variance can be made. Spurious results can arise, as shown in Figure 4, if stationarity is not well established before applying criteria such as Lee-Stone.

A definitive trademark of non-ergodicity is the finding that σ_{mss} does not vanish as the hopping length is increased. In fact, this quantity increases with the hopping length for both the unitary and the symplectic ensemble. Such a conclusion is also supported by the computation of the correlation function, which additionally offers a physical explanation for non-ergodicity: The decay of the correlations in ΔB is such that only a few correlations fields occur within ϕ_0/ℓ^2. As a new effective sample is defined by such a correlation range, only a few of them are defined. Thus field fluctuations are mostly correlated fluctuations.

The experiments of reference [5] were done in the absence of SO. Therefore, in order to make a rigorous comparison with our results one has to check the experimentally considered interval $[B_s(t),\ B_f]$. According to Figure 4a, if one extrapolates data to hopping distances of the order of $t = 10\xi$, the incidence of the averaged field range can reach 20% (width of initial field range indicated) of the absolute value of the fluctuations. On the other hand, the scaling of the disorder fluctuations with the hopp length t, as shown in Figure 4, agrees qualitatively with the experimental results obtained by Orlov *et al.* [26] *i.e.* the fluctuations first increase rapidly ($\mathrm{Var}(\ln|J|)$ faster than t) and then settles on almost linear behavior (see comments in Ref. [2] of Kramer and McKinnon). It should be pointed out that the variance over disorder should converge to $t^{2/3}$ for large enough t [10]. It is interesting to observe that both the Var_d and Var_B appear to have the same functional form when the appropriate field interval is considered. In the case with SO, experimental studies of fluctuations are lacking. To our knowledge, the only work where field fluctuations in the DIR have been reported in samples with SO is the work of Hernandez and Sanquer [27]. Our predictions should give a clear qualitative way of differentiating the unitary and symplectic ensembles in the experimental data.

From the theoretical point of view, we have derived two important objects: the *cooperon* and the *diffuson* which are the weak localization counterparts in the DIR (involving only directed paths). They allow us to explain some features of the correlation function like the qualitative behavior for small ΔB, and numerically, the weakly decaying behavior on the scale (ϕ_0/ℓ^2), that according to the Slutski's theorem, is responsible for the non-ergodic behavior as we have found. One can also account for the slower decay (by a factor of two) of the correlation functions in the symplectic ensemble as a symmetry feature.

In order to experimentally observe persistent diffuson fluctuations, one has to explore a range of parameters so that there is a saturation in the average behavior while the wave function shrinkage [16] is still a negligible effect. This range can be defined by the condition $B_c < \hbar/(ea_B N^{1/3}) = B_{orb}$, where a_B is the Bohr radius. B_{orb} is the scale for the orbital shrinkage to be important [4], *i.e.*, when the cyclotron radius becomes of the order of the mean free path ℓ. These conditions have been met in references [4] and [5]. Furthermore, according to references [3–5] the magnetic field cannot induce geometric fluctuations due to changes in the identity of the hop [28]. This finding holds in both two and three dimensions, and therefore we expect the insulating cooperon and diffuson fluctuations should be seen experimentally also in three dimensions. In experiments, one should be aware about the condition of mesoscopic sample, discussed in Section 3. Otherwise, trivial self-averaging of spatially different oriented hops can wash out the possibility to observe fluctuations.

There will be a non-ergodic to ergodic crossover when one relaxes the condition of DIR with $\ell \ll \xi < t$ such that there are many impurities within ξ. In this case there will be a diffusing behavior within the length scale ξ so that two overlapping random processes are at work, one ergodic for the diffusing scale ξ and the other non-ergodic for the larger scale t [15]. Finally, we want to stress that the fundamental point of considering correlations in the random processes $F(\mathcal{H}, t)$ due to the path intersections cannot be relaxed. This approach permitted us to use the strength of the replica theory, leading to semi-quantitative and qualitative predictions. Theories like the independent path approximation, where such crossings are neglected, miss the very crucial description of fluctuations completely.

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